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Verification of the proposed equation for calculation of the forces on a sphere accelerating in a viscous fluid

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The equation which was proposed for the calculation of forces on a sphere accelerating in a viscous fluid is verified by comparing the velocities of spheres falling under gravity in a viscous fluid with those calculated from the equation.

1. Comparison of calculated and experimental values

Odar & Hamilton (1964) proposed an equation for the force exerted by a viscous fluid on a sphere which is accelerating arbitrarily and moving rectilinearly in an otherwise quiet fluid. The equation was

$$-F = \tfrac{1}{2}C_V \pi R^2 \rho\, |V|\, V + C_A(\tfrac{4}{3}\pi R^3)\rho a + C_H R^2(\pi\rho\mu)^{\frac{1}{2}}\int_0^t \frac{a(t')}{(t-t')^{\frac{1}{2}}}\,dt'. \tag{1}$$

The coefficients, C_A and C_H, were determined empirically from the results of experiments in which the motions of the sphere were simple harmonic with their frequency and amplitude varying between 17 and 159 rev./min, and 1 and 4 in., respectively. The values can be formulated as

$$C_A = 1{\cdot}05 - \frac{0{\cdot}066}{Ac^2+0{\cdot}12}, \tag{2}$$

and

$$C_H = 2{\cdot}88 + \frac{3{\cdot}12}{(Ac+1)^3}, \tag{3}$$

in which $Ac = V^2/aD$ is the acceleration number.

Equation (1) as applied to the problem of the free fall of a sphere in a viscous fluid is solved by using two different methods:† one by successive approximations and the other by simulation with an analogue computer. The results were compared with the experimental data for 42 runs† taken by Moorman (1955). Figure 1 shows two of these runs. The apparent discrepancy in run no. 1 is due to the lack of synchronization of the mechanism which released the sphere in a tank full of fluid, with the Strobotac–Strobolux equipment which measured the descent of the sphere by taking pictures at Δt time increments. Thus, in the first displacement, an error of a considerable fraction of a full time increment occurred in most of the runs, resulting in a shift of the data parallel to the time axis. The

† The reader who is interested in details of computation and comparison of experimental and calculated values for all of the runs is referred to U.S.A. Cold Regions Research and Engineering Laboratory Research Report 190.

initial acceleration, which is calculated from Newton's formula by taking $C_A = \frac{1}{2}$, clearly shows this error. Taking this into consideration, the values calculated from equation 1 show an excellent agreement with the data.

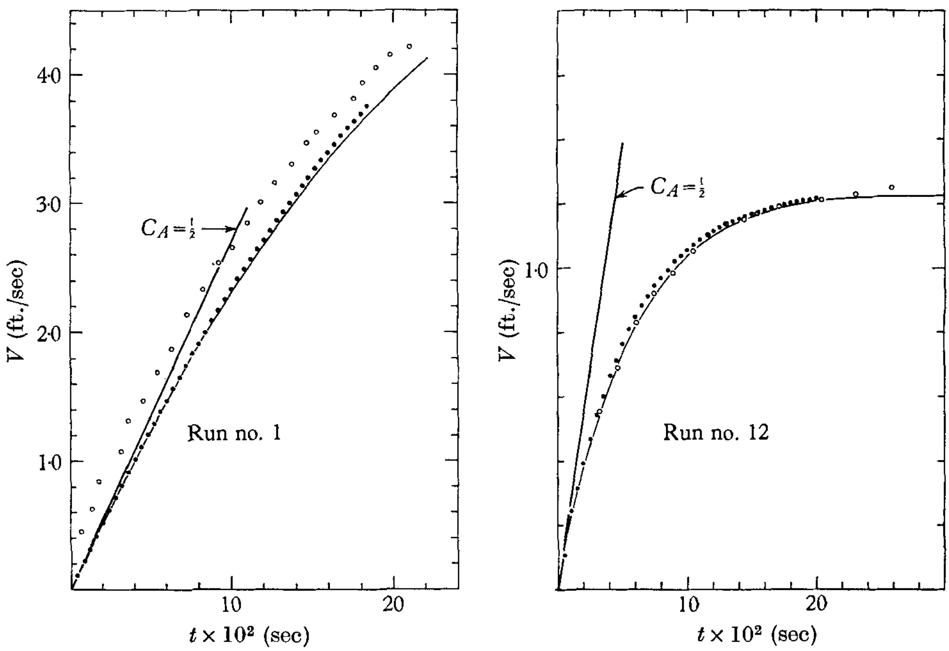

FIGURE 1. Comparison of the calculated values with experimental data. ●, Values calculated by digital computer; ○, values from Moorman's data; —, analogue computer solution.

2. Conclusion

The agreement between the calculated and experimental values of the velocities clearly demonstrates that the proposed equation (1), which was valid for simple harmonic motion, is also valid for free fall. It should be emphasized that a simple harmonic motion and a free fall are very different motions. This suggests that equation (1) has a wide range of application.

REFERENCES

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